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The Synthesis of Variable Structure System for the Control of Quadrotor Spatial Motion

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Abstract. The method of the synthesis of multi-channel decentralized variable structure system for the control of quadrotor spatial motion is developed in this paper. The control law based on the formation of the sliding mode in the separate subsystems of each coordinates control is formed. Each subsystem includes two loops for Cartesian coordinates and relevant Euler angles control. The conditions of the existence of stable sliding mode are obtained. These conditions take into account essential dynamic reciprocal effect between all control channels. The application of proposed control law provides high control quality and maximal fast-action at any variations of the quadrotor parameters within given ranges. This control law does not require the identification of changing object parameters. Efficiency of synthesized control system is confirmed by numerical simulation results. The control system can be realized in the real time by simple technical means.

Introduction

The problem of control of spatial motion of autonomous flying robots (quadrotors) is one of the most actual problem in the modern control theory [1, 2]. The mathematical models of these objects dynamics are characterized by the high dimensionality, nonlinearity and multiple connectivity. It caused by the presence of reciprocal effect between the separate channels of motion control for each degree of freedom with a simultaneous change of several coordinates. Furthermore, the parameters of these models are not often priori determined and its can change over wide limits in the process of motion (because of the instability of mass-inertia characteristics of quadrotors, for example). These factors essentially complicate the solution of the synthesis task.

At present the variable structure systems (VSS) are very successfully used for the control of non-stationary dynamic objects [3, 4]. These systems provide the high quality indicators and robustness with respect to the variable parameters of the control object by means of the specially organized sliding mode [5]. In this case the required motion of object is ensured due to the matched formation of separate assigning actions for each control channel of its any coordinates. The methods of VSS synthesis for the nonlinear objects are proposed by [6-8].

The construction of the systems of decentralized control with the use of preliminary decomposition of the mathematical models of these objects is one of the most common approaches to the VSS synthesis [9]. However, such control systems for the quadrotors are not successfully developed now.

In this connection the synthesis of multi-channel decentralized variable structure system for the control of quadrotor is the purpose of present paper. The control law based on the formation of the sliding mode in the separate subsystems is developed. This control law allows compensating any variations of the object parameters (in the given limits) without its identification. The necessary and sufficient conditions of stable existence of the indicated sliding mode are determined and proved with the presence of cross couplings between the internal coordinates (essential dynamic reciprocal effect between all subsystems). The efficiency of synthesized control system is confirmed by numerical simulation results.

Features of Quadrotor Mathematical Model

The dynamics of quadrotor as a free rigid body moving (at air resistance present) due to the thrust forces generated by the four rotating propellers is determined by the system of nonlinear non-stationary differential equations in the matrix-vector form [10].

For convenience of further synthesis of the quadrotor decentralized control system we shall present its mathematical model as a system of six nonlinear non-stationary differential equations in scalar form:

$$\begin{aligned}
\dot{\upsilon}_x &= \upsilon_y\omega_z - \upsilon_z\omega_y + g\sin\theta - k_x\upsilon_x^2\,\mathrm{sign}\,\upsilon_x / m\,,\\
\dot{\upsilon}_y &= \upsilon_z\omega_x - \upsilon_x\omega_z - g\cos\theta\sin\varphi - k_y\upsilon_y^2\,\mathrm{sign}\,\upsilon_y / m\,,\\
\dot{\upsilon}_z &= \upsilon_x\omega_y - \upsilon_y\omega_x - g\cos\theta\cos\varphi + f_1/m - k_z\upsilon_z^2\,\mathrm{sign}\,\upsilon_z / m\,,\\
\dot{\omega}_x &= \omega_y\omega_z(J_y - J_z)/J_x - \omega_y\Omega_d J_m/J_x + f_2 l/J_x\,,\\
\dot{\omega}_y &= \omega_x\omega_z(J_z - J_x)/J_y + \omega_x\Omega_d J_m/J_y + f_3 l/J_y\,,\\
\dot{\omega}_z &= \omega_x\omega_y(J_x - J_y)/J_z + f_4/J_z\,,
\end{aligned} \qquad (1)$$

where, υ_x, υ_y, υ_z are projections of the quadrotor linear velocity vector $\boldsymbol{\upsilon}$ on the axes of vehicle-fixed reference frame, ω_x, ω_y, ω_z are projections of the quadrotor angular velocity vector $\boldsymbol{\omega}$ on the axes of vehicle-fixed reference frame, φ, θ are the pitch and roll angles, Ω_d is a total angular velocity (taking into account the direction of rotation) of the four motors rotation, f_1 is a total thrust providing the height change, f_2 is a difference between the thrusts providing the roll angle change, f_3 is a difference between the thrusts providing the pitch angle change, f_4 is a total moment providing the yaw angle ψ change, m is a quadrotor mass, J_x, J_y, J_z are quadrotor inertia moments relative to the axes of the vehicle-fixed reference frame, J_m is a total inertia moment of the each thruster rotating parts (rotor and propeller), k_x, k_y, k_z are drag coefficients of air resistance when quadrotor driving along the corresponding axes of the vehicle-fixed reference frame, l is a distance between the vertical axis z of the vehicle-fixed reference frame and the axis of each rotor rotation, g is an acceleration of gravity.

In accordance with the quadrotor design features and operation principle (including quadratic character of dependence between the motor thrust and the reactive moment of air resistance and the angular velocity of its rotation), the expressions for Ω_d and f_1, f_2, f_3, f_4 have the follow form:

$$\Omega_d = -\Omega_1 + \Omega_2 - \Omega_3 + \Omega_4\,, \qquad (2)$$

$$\begin{aligned}
f_1 &= b(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)\,,\\
f_2 &= b(-\Omega_1^2 - \Omega_2^2 + \Omega_3^2 + \Omega_4^2)\,,\\
f_3 &= b(-\Omega_1^2 + \Omega_2^2 + \Omega_3^2 - \Omega_4^2)\,,\\
f_4 &= d(-\Omega_1^2 + \Omega_2^2 - \Omega_3^2 + \Omega_4^2)\,,
\end{aligned} \qquad (3)$$

where, Ω_1, Ω_2, Ω_3, Ω_4 are angular velocities of each propeller rotation, b is a coefficient of each propeller thrust, d is a drag coefficient of air resistance for each propeller rotation.

The quadrotor parameters m, J_x, J_y, J_z, k_x, k_y, k_z can change during its operation, depending on the payload value, moving modes and characteristics of the environment.

The connection between a quadrotor linear and angular velocities projections in the earth-fixed and vehicle-fixed reference frames is defined by following equations:

$$\begin{aligned}
\dot{x} &= \upsilon_x\cos\psi\cos\theta + \upsilon_y(\cos\varphi\sin\psi - \sin\varphi\cos\psi\sin\theta) + \upsilon_z(\sin\varphi\sin\psi + \cos\varphi\cos\psi\sin\theta)\,,\\
\dot{y} &= \upsilon_x\sin\psi\cos\theta + \upsilon_y(\cos\varphi\cos\psi + \sin\varphi\sin\psi\sin\theta) - \upsilon_z(\sin\varphi\cos\psi - \cos\varphi\sin\psi\sin\theta)\,,\\
\dot{z} &= -\upsilon_x\sin\theta + \upsilon_y\sin\psi\cos\theta + \upsilon_z\cos\varphi\cos\theta\,,\\
\dot{\varphi} &= \omega_x + \omega_y\sin\varphi\,\mathrm{tg}\,\theta + \omega_z\cos\varphi\,\mathrm{tg}\,\theta\,,
\end{aligned} \qquad (4)$$

$$\dot{\theta} = \omega_y \cos\varphi - \omega_z \sin\varphi,$$
$$\dot{\psi} = \omega_y \sin\varphi / \cos\theta + \omega_z \cos\varphi / \cos\theta .$$

Taking into account the inertia of each thruster, we shall present its mathematical model as follows:

$$T_f \dot{F}_i + F_i = k_f g_i, \tag{5}$$

where, F_i is a thrust force of i-th thruster (here i = 1, 2, 3, 4), g_i is a control signal for i-th thruster, T_f, k_f are time constant and gain of each thruster.

Further we shall solve the problem of the synthesis of decentralized control system for the multidimensional and multiply connected nonlinear dynamic object (written by equations from 1 to 5), which will provide high accuracy and control processes quality at any variations of its parameters in given ranges. We shall find this solution in the class of VSS. These control systems provide the invariance to quadrotor parameters changes by the sliding mode organization. Parameters of this sliding mode will define the desired process of object motion.

The Development of a Generalized Structure of Quadrotor Control System

In accordance with the principle of decentralized control we shall present the quadrotor control system in whole as a set of separate subsystems (control channels). Each of these subsystems provides an individual control for one of its spatial coordinates. In this case, we shall take into account that a method of providing of quadrotor space motion presupposes movement in a horizontal plane due to changes of roll and pitch angles only. Therefore, we shall build motion control subsystems on the x and y coordinates on the principle of the subordinate regulation. Each subsystem must include two control loops: the loop for coordinates control (external loop) and the loop for the relevant Euler angles control (internal loop).

Required decomposition, i.e. compensation of relationships between subsystems and independence for each coordinate control, will be provided by a special block for control actions conversion. The inputs of this block are g_1, g_2, g_3 and g_4 signals from the outputs of the height regulator and roll, pitch, yaw angles regulators. In accordance with Eq. 3 the thrusters control signals are formed as follows:

$$\begin{aligned} u_1 &= g_1 - g_2 - g_3 - g_4, \\ u_2 &= g_1 - g_2 + g_3 + g_4, \\ u_3 &= g_1 + g_2 + g_3 - g_4, \\ u_4 &= g_1 + g_2 - g_3 + g_4. \end{aligned} \tag{6}$$

The structural scheme of proposed control system of quadrotor motion is shown in Fig. 1. Here x_d, y_d, z_d, ψ_d are desirable values of coordinates. The standard linear regulators of x and y coordinates can be used in the external loops if high quality robust control devices are used in the internal loops.

Synthesis of VSS-Controllers in the Separate Channels of Quadrotor Control

As mentioned above, we shall synthesize VSS-regulator in each subsystem (control channel) on the basis of the theory of variable structure systems in order to ensure the robustness of the quadrotor control system in relation to possible changes of object parameters and external influences.

Considering the decomposition of the quadrotor control system, we shall make synthesis separately for each subsystem. For example, consider the channel (subsystem) for quadrotor height (coordinate z) control.

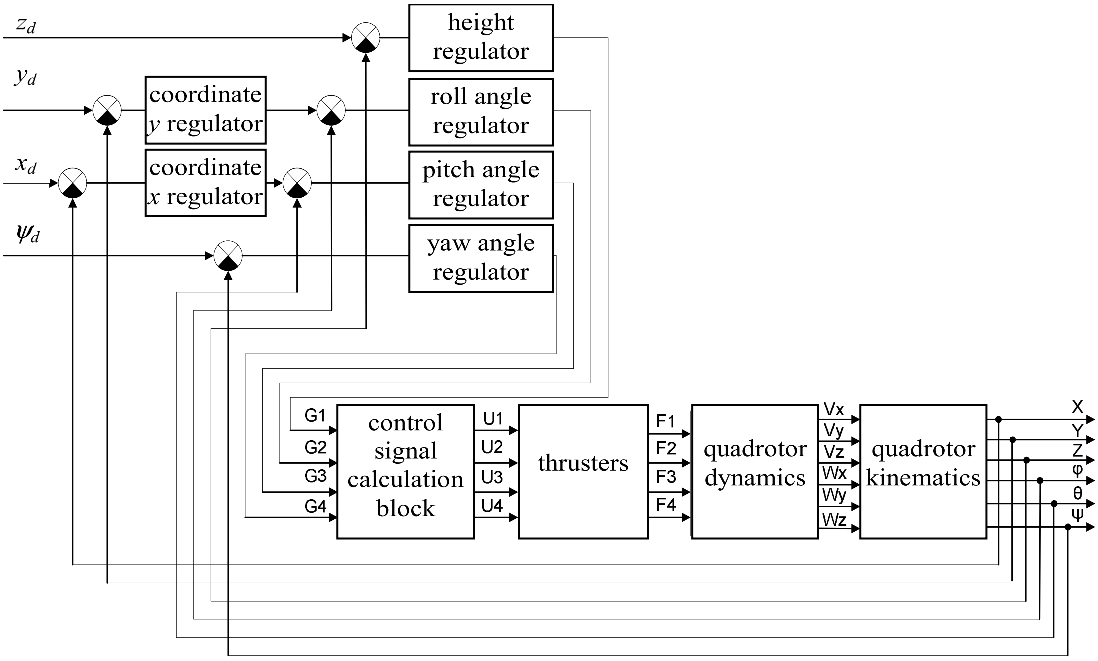

Fig. 1 Generalized structural scheme of quadrotor control system.

We shall allocate additional control loops for the coordinate z and the velocity υ_z regulation in this subsystem, as shown in Fig. 2. We shall use a standard linear regulator in the external loop and VSS-regulator in the internal loop.

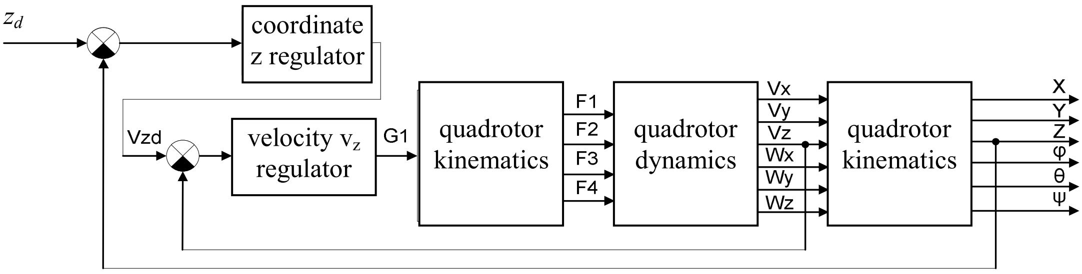

Fig. 2 Structural scheme of quadrotor height control subsystem.

We shall assume the control is carried out on coordinate z only. In this case $\upsilon_x = \upsilon_y = \omega_x = \omega_y = 0$ and $\cos\theta = \cos\varphi = 1$. In a view of these assumptions the third equation of system (1) takes the form:

$$m\dot{\upsilon}_z + b_z\upsilon_z^2 + mg = f_1\,, \tag{7}$$

where, $b_z = k_z \,\mathrm{sign}\, \upsilon_z$ is an auxiliary coefficient.

Let the desirable process of υ_z velocity change is described by a linear differential equation of the first order with constant coefficients k_υ and k'_υ:

$$\dot{\upsilon}_z + k_\upsilon\upsilon_z = \dot{\upsilon}_{zd} + k'_\upsilon\upsilon_{zd}\,, \tag{8}$$

where, υ_{zd} is a desirable value of υ_z velocity.

We shall pose the problem to synthesize a such g_1 control law, in which the behavior of the system described by Eq. 5 and Eq. 7 would be consistent with Eq. 8 for any control object parameters in a given ranges and external perturbations. It is necessary to ensure the existence of a sliding mode in order to solve this problem. The control system behavior in this mode is described by the first-order equation, the solution of which will depend on the selected parameter k_s only, not by Eq. 5 and Eq. 7:

$$\dot{e} + k_s e = 0\,, \tag{9}$$

where, k_s is a sliding line coefficient, $e = \upsilon_{zd} - \upsilon_z$ is an error on υ_z velocity.

As known, the condition of the sliding mode existence is described by the following inequality:

$$s\dot{s} < 0\,, \tag{10}$$

where, $s = \dot{e} + k_s e$ is a linear combination of error and its derivative.

Thus, it is necessary to provide the Ineq. 10 fulfillment by the control law g_1 for the system described with Eq. 5, Eq. 7 in order to solve this problem. After a set transformations of these equations, we shall obtain the required control law in VSS in the follow view:

$$g_1 = W \operatorname{sign} s\,, \tag{11}$$

where, $W > 0$ is a function that determines the amplitude of the relay control signal.

It is easy to prove that the condition (Ineq. 10) of the sliding mode existence fulfill for all values of the quadrotor parameters in given ranges and external perturbations if we shall select the W function:

$$W = k_{u1}\,|e| + k_{u2} e^2 + k_{u3}\,|f_1|\,, \tag{12}$$

where, $k_{u1} > 0$, $k_{u2} > 0$, $k_{u3} > 0$ are constant regulator coefficients.

The conditions of these coefficients selection may be represented by the following inequalities:

$$k_{u1} > \max T_f m\,|k_t((1/T_f + k_s)\upsilon_{zd} - \dot{\upsilon}_{zd}) - k_s(1/T_f + k_s) + k_f k_{u3}(k_s + k_t \upsilon_{zd})/T_f\,|/k_f\,, \tag{13}$$

$$k_{u2} > \max T_f m\,|k_t((1/2T_f + k_s) + k_f k_t k_{u3}/2T_f\,|/k_f\,, \tag{14}$$

$$k_{u3} > \max T_f\,|\dot{f}/f + 1/T_f\,|/k_f\,, \tag{15}$$

where, $k_t = 2b_z / T_f m$, $f = m\dot{\upsilon}_{zd} + b_z \upsilon_{zd}^2 + mg$ are auxiliary coefficient and function.

Thus, the control system will be operating in the sliding mode after entering the image point to the sliding line. The system dynamics will describe by Eq. 8 with the coefficients $k_\upsilon = k'_\upsilon = k_s$, by using the obtained control law (Eq. 11-15).

As a numerical simulation results shown required control quality and matching of the processes in the control system with the desirable process for any quadrotor parameters in the given ranges is provided by using of synthesized VSS-controller.

Conclusions

The new method of the synthesis of multi-channel variable structure system for the decentralized control of the quadrotor is proposed in this paper. The conditions of the existence of stable sliding mode with the presence of essential dynamic reciprocal effect between all control channels are obtained. This control law does not require the identification of changing object parameters. The application of new synthesis method made it possible to ensure the high control quality at any variations of the object parameters within given ranges. The control laws proposed can be realized in the real time by simple technical means.

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